Benha University
Faculty Of Engineering at Shoubra

ECE 411
Antennas & Wave propagations
(2016/2017)
Lecture (4)
Antenna Parameters
Point Sources

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Agenda

- Remember (Antenna Parameters)
- Polarization
- Examples
- Chapter (3) : Point sources

1 - Remember (Antenna parameters)

Remember (Solid angle & Directivity)

$$D = \frac{4\pi}{\Omega_A}$$

Exact

$$\Omega_A = \iint_{4\pi} P_n(\theta,\varphi)\,d\Omega = \iint_{4\pi} U_n(\theta,\varphi)\,d\Omega = \iint_{4\pi} E_n^2(\theta,\varphi)\,d\Omega$$

$$d\Omega = \sin\theta\, d\theta\, d\phi$$

Approximate (Sr)

$$\Omega_A = \Theta_{HP}(rad) * \Phi_{HP}(rad)$$

Approximate (degree square)

$$D = \frac{41253}{\Theta^o_{HP} * \Phi^o_{HP}}$$

$$P_{rad} = \oiint_{4\pi} W_{av}\, dA = \oiint_{4\pi} \frac{1}{2}\mathrm{Re}(E \chi H^{*}).(r^{2} Sin\theta d\theta d\phi) = \oiint_{4\pi} U\, d\Omega = \frac{1}{2} I_o^{2} R_r$$

$$P_{rad} = e_t * P_{in}$$

$$U = r^{2} * W_{av}$$

$$D = \frac{4\pi}{\Omega_A} = \frac{4\pi}{\iint_{4\pi} U_n(\theta,\varphi) d\Omega} = \frac{4\pi U_{\max}}{\iint_{4\pi} U(\theta,\varphi) d\Omega} = \frac{4\pi U_{\max}}{P_{rad}}$$

$$e_t = e_r e_{cd}$$

$$e_{cd} = \frac{R_r}{R_L + R_r} \quad \text{(dimensionless)}$$

$$e_r = 1 - |\Gamma|^{2}$$

$$\Gamma = \frac{Z_{input} - Z_{generator}}{Z_{input} + Z_{generator}}$$

$$G = e_t \cdot D = e_r e_{cd} \cdot D$$

$$e_t \leq 1$$

$$R_{rad} = \frac{2P_{rad}^{total}}{|I_o|^2} = \frac{2\oiint_{4\pi} U(\theta,\varphi)d\Omega}{|I_o|^2}$$

For Infinitesimal Dipole

$$R_{rad} = 80\pi^2 (\frac{\ell}{\lambda})^2$$

For Short Dipole

$$R_{rad} = 20\pi^2 (\frac{\ell}{\lambda})^2$$

$$D = \frac{4\pi}{\Omega_A} = 4\pi \frac{A_e}{\lambda^2}$$

$$A_{em} = \frac{\lambda^2}{4\pi} D_o$$

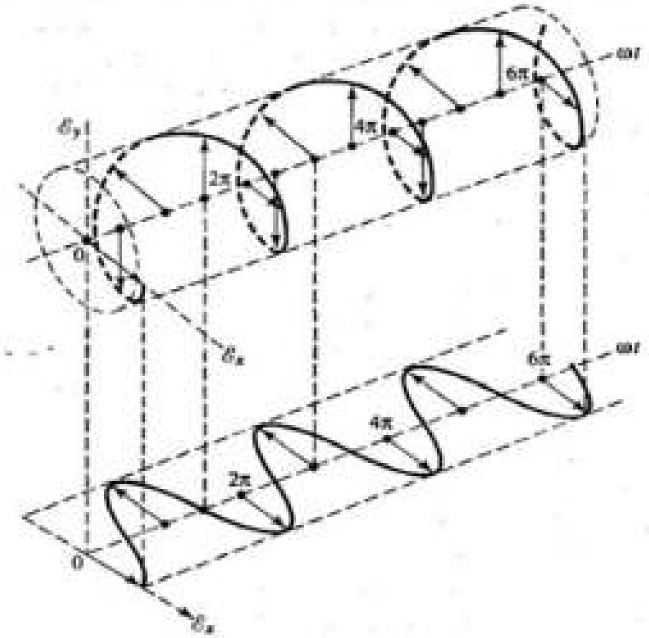
$$A_e = \xi_{ap} A_{em}$$

$$\frac{P_r}{P_t} = \left(\frac{\lambda}{4\pi R}\right)^2 G_t G_r$$

2 - Polarization

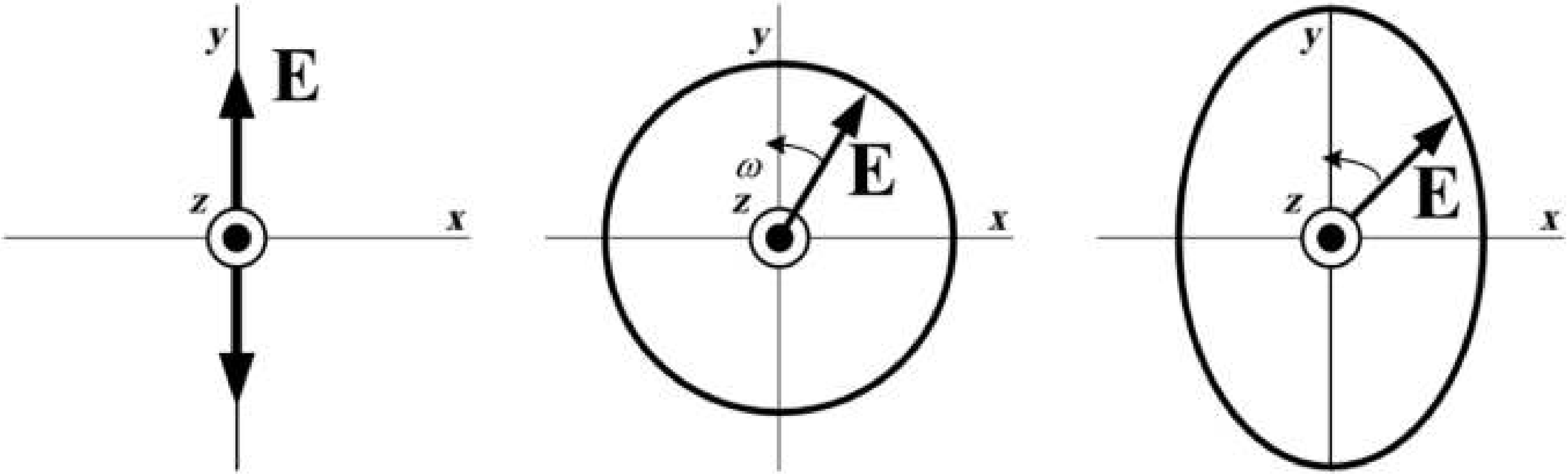
2 - Polarization

The polarization of the EM field describes the orientation of its vectors at a given point and how it varies with time. In other words, it describes the way the direction and magnitude of the field vectors (usually **E**) change in time.



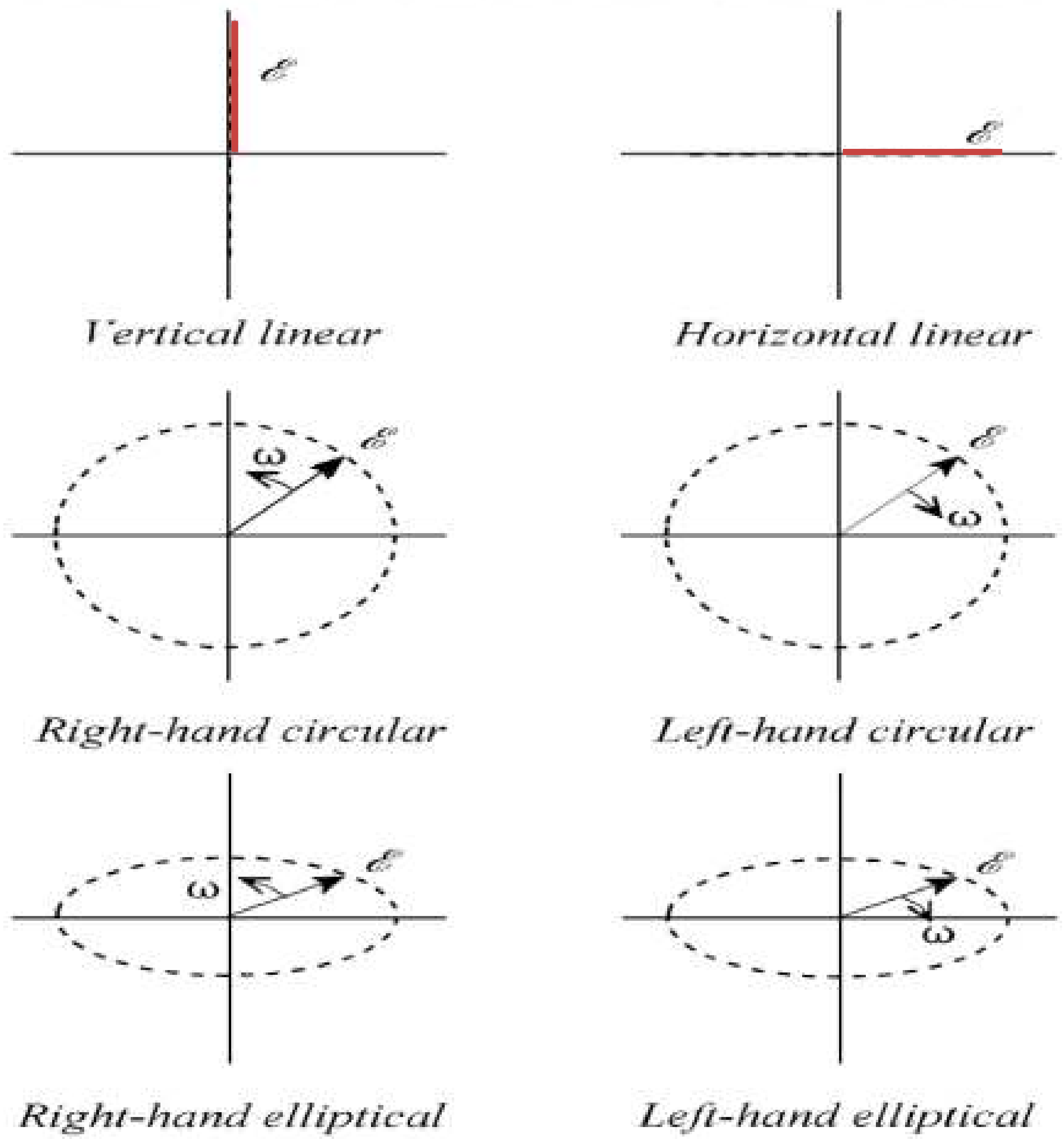
According to the shape of the trace, three types of polarization exist for harmonic fields: ***linear***, ***circular*** and ***elliptical***. Any polarization can be represented by two orthogonal linear polarizations, (E_x, E_y) or (E_H, E_V), the fields of which may, in general, have different magnitudes and may be out of phase by an angle δ_L.

2 - Polarization



(a) linear polarization (b) circular polarization (c) elliptical polarization

2 - Polarization



2 - Polarization

$$E_x = E_{xo} Sin(\omega t - Kz)$$

$$E_y = E_{yo} Sin(\omega t - Kz + \Delta\phi)$$

I. Linear

A. $E_{xo} \neq 0, \quad E_{yo} = 0$

B. $E_{xo} = 0, \quad E_{yo} \neq 0$

C. $E_{xo} \neq 0, \quad E_{yo} \neq 0$

$$\Delta\phi = \pm n\pi, \quad n = 0, 1, 2, \ldots$$

II. Circular

$$E_{xo} = E_{yo} \qquad (2\text{-}59)$$

$$\Delta\phi = \pm\left(\frac{1}{2} + n\right)\pi, \quad n = 0,1,2,\ldots \qquad (2\text{-}60, \text{-}61)$$

\+ : clockwise (RH)

\- : counterclockwise (LH)

III. Elliptical

A. $E_{xo} \neq E_{yo}, \quad \Delta\phi \neq \pm n\pi, \quad n = 0,1,2,\ldots$

B. $E_{xo} = E_{yo}, \quad \Delta\phi \neq \pm\left(\frac{1}{2}+n\right)\pi, \quad n = 0,1,2,\ldots$

(2-62a,b)

$$\text{AR} = \frac{\text{major axis}}{\text{minor axis}} = \frac{OA}{OB} \qquad 1 \leq \text{AR} \leq \infty$$

(2-65)

2 - Polarization

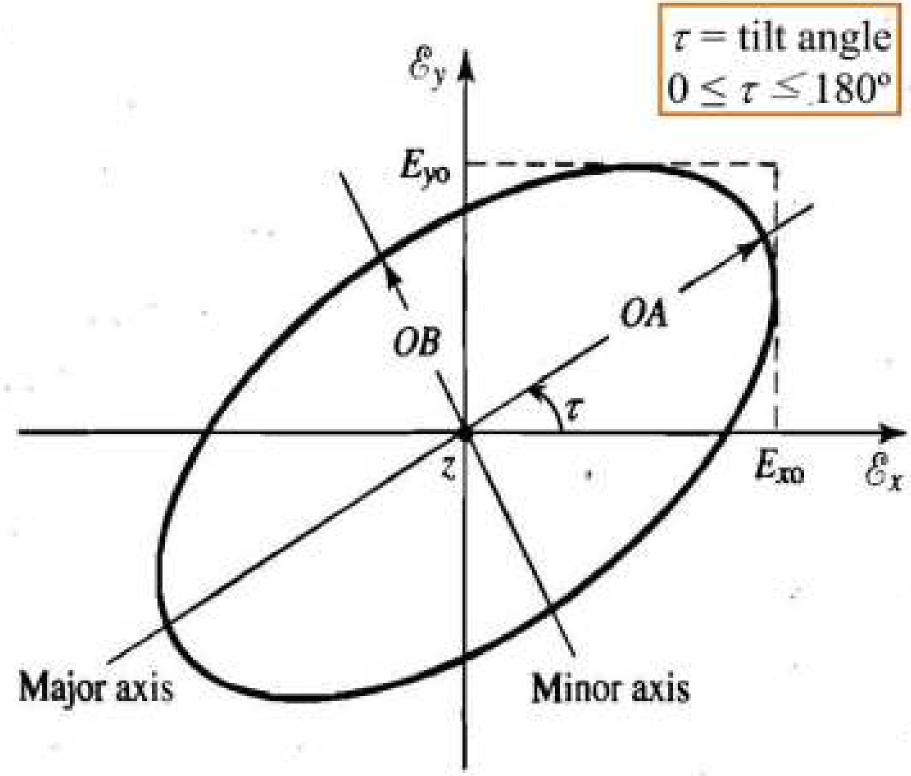
Axial Ratio

The polarization state of an EM wave can also be indicated by another two parameters: Axial Ratio (AR) and the tilt angle (τ). AR is a common measure for antenna polarization. It definition is:

$$AR = \frac{OA}{OB}, \qquad 1 \leq AR \leq \infty, \quad \text{or} \quad 0\ \text{dB} \leq AR \leq \infty \text{dB}$$

where OA and OB are the major and minor axes of the polarization ellipse, respectively. The tilt angle τ is the angle subtended by the major axis of the polarization ellipse and the horizontal axis.

2 - Polarization



Tilt angle

$$\tau = \frac{\pi}{2} - \frac{1}{2}\tan^{-1}\left[\frac{2E_{xo}E_{yo}}{E_{xo}^2 - E_{yo}^2}\cos(\Delta\phi)\right]$$

For example:

$AR = 1, \Rightarrow$ circular polarization

$1 < AR < \infty, \Rightarrow$ elliptical polarization

$AR = \infty, \Rightarrow$ linear polarization

2 - Examples

EX: 1

Two $\lambda/2$ dipoles are crossed at 90°. If the two dipoles are fed with equal currents, what is the polarization of the radiation perpendicular to the plane of the dipoles if the currents are (a) in phase, (b) phase quadrature (90° difference in phase) and (c) phase octature (45° difference in phase)?

***Solution*:**

(a) LP

(b) CP

(c) Ep

EX: 2

A wave travelling normally out of page toward you is resultant of two linearly polarized component $E_x = 3\cos(\omega t)$ and $E_y = 2\cos(\omega t+90)$ find the (i) axial ratio (ii) band of rotation (CW or CCW)

$Ex = 3\cos\omega t$

$Ey = 2\cos(\omega t+90) = 2\cos\omega t \cos 90 - 2\sin\omega t \sin 90$ $(\cos 90=0 \& \sin 90=1)$

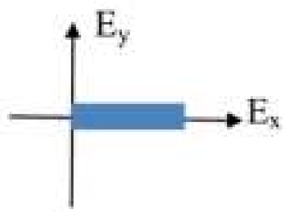
$Ey = -2\sin\omega t$

$\left(\frac{Ex}{3}\right)^2 + \left(\frac{Ey}{-2}\right)^2 = \sin^2\omega t + \cos^2\omega t = 1$ (Elliptical Ep)

$AR = \left|\frac{major}{minor}\right| = \left|\frac{3}{2}\right| = 1.5$

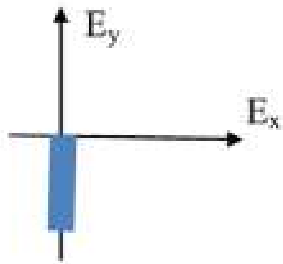
rotation $\omega t = 0$ $Ex = 3\cos 0 = 3$

$Ey = -2\sin 0 = 0$



$\omega t = \frac{\pi}{2}$ $Ex = 3\cos\frac{\pi}{2} = 0$

$Ey = -2\sin\frac{\pi}{2} = -2$



So rotation is clock wise

EX: 3

A wave traveling normally out of the page is resultant two elliptically polarized (EP) waves, one with components $E_x = 5Cos\omega t$ and $E_y=3Sin\omega t$ and another with components $E_r=3e^{j\omega t}$ and $E_L=4e^{-j\omega t}$. For the resultant wave, find (a) AR, and (b) the band of rotation and polarization.

1^st component

$E_{x1}=5Cos\omega t$

$E_{y1}=3Sin\omega t.$

2^nd component

$E_r=3e^{j\omega t}=3Cos\omega t + j3Sin\omega t$

$E_l=4e^{-j\omega t}=4Cos\omega t-j4Sin\omega t$

So $E_{x2}=3Cos\omega t+4Cos\omega t=7Cos\omega t$

$E_{y2}=3Sin\omega t-4Sin\omega t=-Sin\omega t$

Total components

$E_{xt}= E_{x1} + E_{x2}=5Cos\omega t +7Cos\omega t=12 \, Cos\omega t$.

$E_{yt}=E_{y1}+ E_{y2}=3Sin\omega t - Sin\omega t=2Sin\omega t$

So $(\frac{E_{xt}}{12})^2 + (\frac{E_{yt}}{2})^2 = 1$... *Ellipse*

(a) $AR = 12/2=6$

(b) Put $\omega t=0$, 90 , *you will find that this wave is Right polarized & CCW*

EX: 4

A wave traveling normally outward from the page (toward the reader) is the resultant of two elliptically polarized waves, one with components of **E** given by

$$E'_x = 6\cos\left(\omega t + \tfrac{\pi}{2}\right) \quad \textbf{and} \quad E'_y = 2\cos\omega t$$

$E''_x = 3\cos\left(\omega t - \tfrac{\pi}{2}\right)$ and $E''_y = 1\cos\omega t$ and the other with components given by

(a) What is the axial ratio of the resultant wave?

(b) Does **E** rotate clockwise or counterclockwise?

***Solution*:**

$$E_y = E'_y + E''_y = 2\cos\omega t + \cos\omega t = 3\cos\omega t$$

$$E_x = E'_x + E''_x = 6\cos(\omega t + \pi/2) + 3\cos(\omega t - \pi/2) = -6\sin\omega t + 3\sin\omega t = -3\sin\omega t$$

(a) E_x and E_y are in phase quadrature and $\mathrm{AR} = 3/3 = 1$ (CP) (*ans.*)

(b) At $t = 0$, $\mathbf{E} = \hat{\mathbf{y}}3$, at $t = T/4$, $\mathbf{E} = -\hat{\mathbf{x}}3$, therefore rotation is CCW (*ans.*)

EX: 5

An elliptically polarized wave traveling in the positive z direction in air has x and y components:

$$E_x = 3\sin(\omega t - \beta x) \qquad (\mathrm{V\,m^{-1}})$$
$$E_y = 6\sin(\omega t - \beta x + 75°) \qquad (\mathrm{V\,m^{-1}})$$

Find the average power per unit area conveyed by the wave.

Solution

The average power per unit area is equal to the average Poynting vector, which from (3) has a magnitude

$$\mathbf{S}_{av} = \frac{1}{2}\frac{E^2}{Z} = \frac{1}{2}\frac{E_1^2 + E_2^2}{Z}$$

From the stated conditions, the amplitude $E_1 = 3\ \mathrm{V\,m^{-1}}$ and the amplitude $E_2 = 6\ \mathrm{V\,m^{-1}}$. Also for air $Z = 377\ \Omega$. Hence

$$\mathbf{S}_{av} = \frac{1}{2}\frac{3^2 + 6^2}{377} = \frac{1}{2}\frac{45}{377} \approx 60\ \mathrm{mW\,m^{-2}} \qquad \textit{Ans.}$$

Chapter (3) : point Sources

1 - Isotropic Source

A POWER THEOREM[1] AND ITS APPLICATION TO AN ISOTROPIC SOURCE. If the Poynting vector is known at all points on a sphere of radius r from a point source in a lossless medium, *the total power radiated by the source is the integral over the surface of the sphere of the radial component S_r of the average Poynting vector*. Thus,

$$P = \oiint \mathbf{S} \cdot \mathbf{ds} = \oiint S_r \, ds$$

where P = power radiated, W

S_r = radial component of average Poynting vector, W m^{-2}

ds = infinitesimal element of area of sphere (see Fig. 3-2*b*)

$= r^2 \sin\theta \, d\theta \, d\phi$, m^2

For an isotropic source, S_r is independent of θ and ϕ so

$$P = S_r \oiint ds = S_r \times 4\pi r^2 \qquad \text{(W)}$$

and

$$S_r = \frac{P}{4\pi r^2} \qquad (\text{W m}^{-2})$$

$r^2 S_r = U$ = radiation intensity $\qquad P = 4\pi U_0 \qquad$ (W)

2 – Source with hemispheric power pattern

the radiation intensity equals a constant U_m in the upper hemisphere and is zero in the lower hemisphere, Then, the total power radiated is the radiation intensity integrated over a hemisphere, or

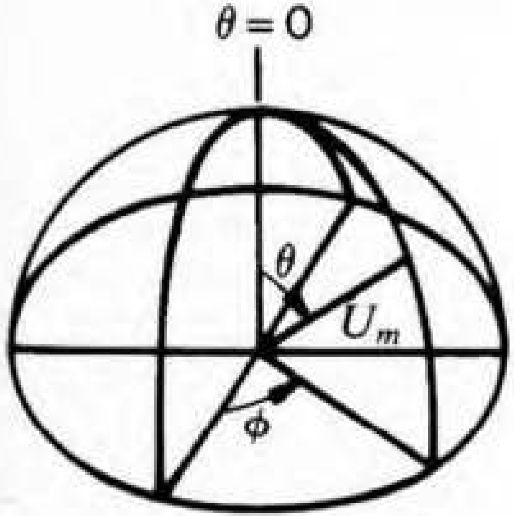
$$P = \iint U \, d\Omega = \int_0^{2\pi} \int_0^{\pi/2} U_m \sin\theta \, d\theta \, d\phi = 2\pi U_m$$

Assuming that the total power P radiated by the hemispheric source is the same as the total power radiated by an isotropic source taken as a reference,

$$2\pi U_m = 4\pi U_0$$

or

$$\frac{U_m}{U_0} = 2 = \text{directivity}$$



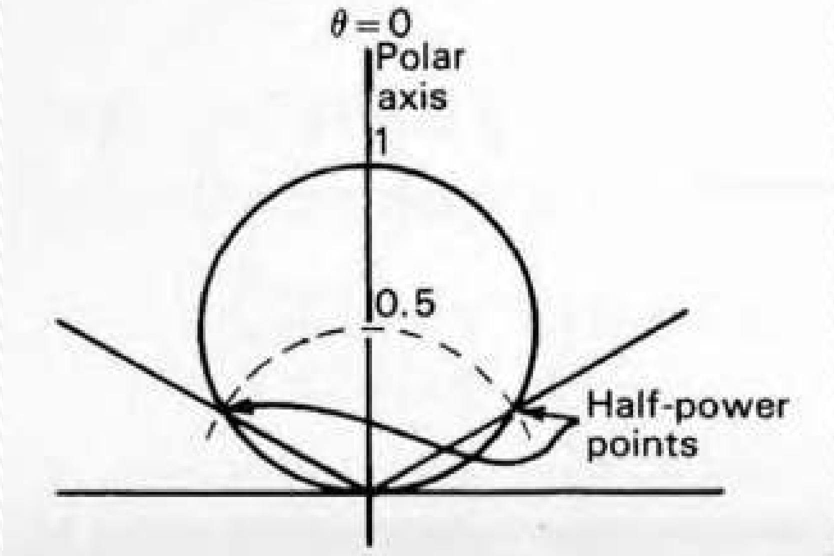
3 – Source with Unidirectional cosine power pattern

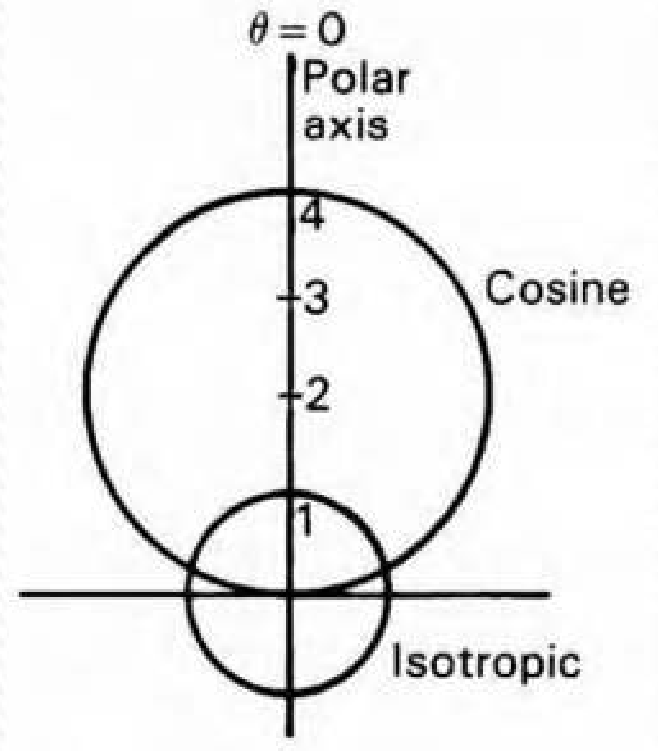
$$U = U_m \cos\theta$$

$$P = \int_0^{2\pi}\int_0^{\pi/2} U_m \cos\theta \sin\theta \, d\theta \, d\phi = \pi U_m$$

$$\pi U_m = 4\pi U_0$$

$$\text{Directivity} = \frac{U_m}{U_0} = 4$$





4 – Source with Bidirectional cosine power pattern

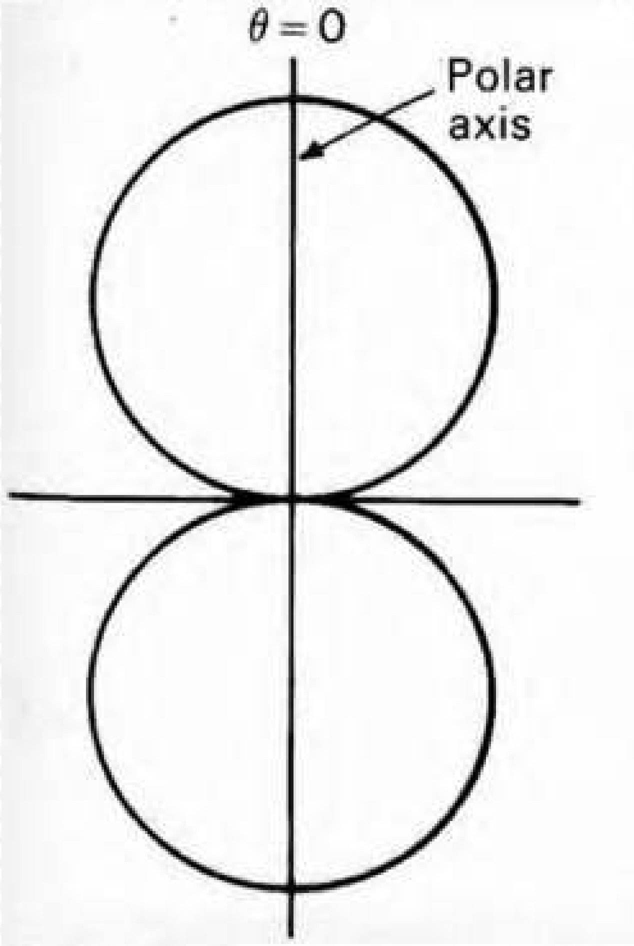
P is twice its value for the unidirectional cosine power pattern, and hence the directivity is 2 instead of 4.

$$U = U_m \cos\theta$$

$$P = 2\int_0^{2\pi}\int_0^{\pi/2} U_m \cos\theta \sin\theta \, d\theta \, d\phi = 2\pi U_m$$

$$2\pi U_m = 4\pi U_0$$

$$\text{Directivity} = \frac{U_m}{U_0} = 2$$



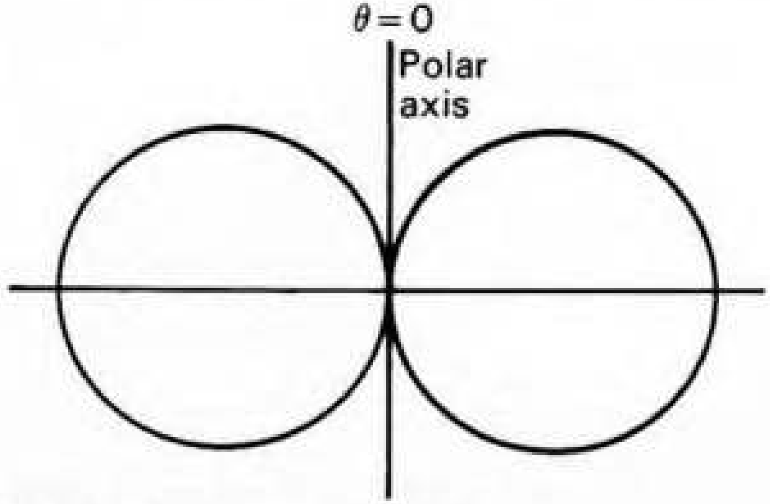
5 – Source with Sine (Doughnut) power pattern

$$U = U_m \sin\theta$$

$$P = U_m \int_0^{2\pi}\int_0^{\pi} \sin^2\theta \, d\theta \, d\phi = \pi^2 U_m$$

$$\pi^2 U_m = 4\pi U_0$$

$$\text{Directivity} = \frac{U_m}{U_0} = \frac{4}{\pi} = 1.27$$



Sine power pattern.

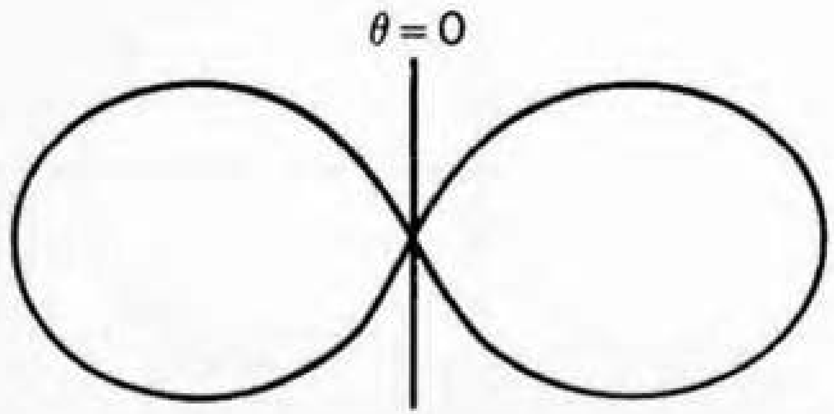
6 – Source with Sine-squared power pattern

$$U = U_m \sin^2 \theta$$ short dipole

$$P = U_m \int_0^{2\pi} \int_0^{\pi} \sin^3 \theta \, d\theta \, d\phi = \tfrac{8}{3}\pi U_m$$

$$\tfrac{8}{3}\pi U_m = 4\pi U_0$$

$$\text{Directivity} = \frac{U_m}{U_0} = \frac{3}{2} = 1.5$$



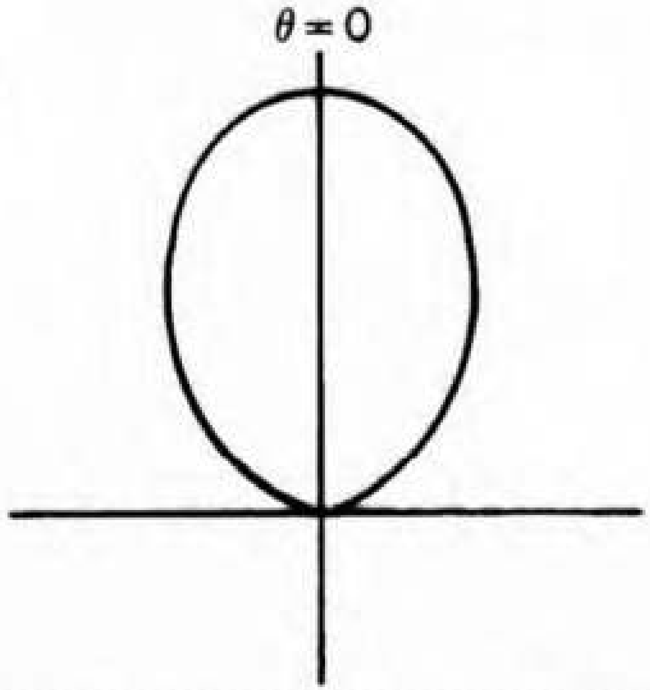
7 – Source with Unidirectional Cosine-squared power pattern

$$U = U_m \cos^2 \theta$$

$$P = U_m \int_0^{2\pi} \int_0^{\pi/2} \cos^2 \theta \sin \theta \, d\theta \, d\phi = \tfrac{2}{3}\pi U_m$$

$$\tfrac{2}{3}\pi U_m = 4\pi U_0$$

$$\text{Directivity} = \frac{U_m}{U_0} = 6$$

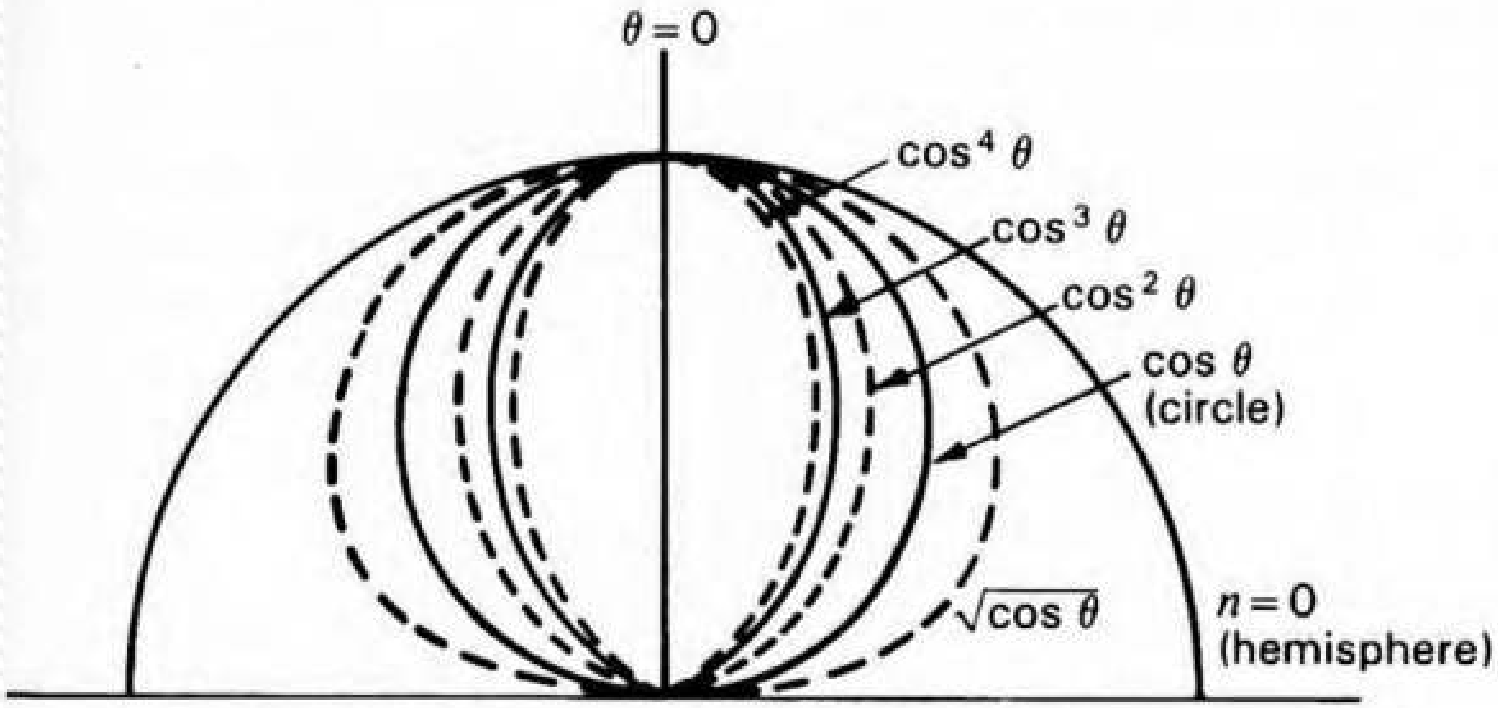


(football shape)

8 – Source with Unidirectional Cosine^n power pattern

$$U = U_m \cos^n \theta$$

$$\text{directivity} = 2(n+1).$$



Unidirectional $\cos^n \theta$ power patterns for various values of n.

Numerical Integration

Trapezoidal Rule

$$\int_a^b f(x)dx = \frac{b-a}{N}\left(\left(\frac{f_o + f_N}{2}\right) + f_1 + f_2 + f_3 + \ldots\ldots + f_{N-1}\right)$$

QUIZ

Quiz

Directivity. Show that the directivity for a source with a unidirectional power pattern given by $U = U_m \cos^n \theta$ can be expressed as $D = 2(n + 1)$. U has a value only for $0 \leq \theta \leq \pi/2$ and $0 \leq \phi \leq 2\pi$ and is zero elsewhere.

Next Lecture (5)

Chapter(4): Arrays of point Sources

Two Isotropic point sources & nonisotropic

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Thank You